2nd Grade Common Core State Standards

Flip Book

This document is intended to show the connections to the Standards of Mathematical Practices for the content standards and to get detailed information at each level. Resources used: CCSS, Arizona DOE, Ohio DOE and North Carolina DOE. This "Flip Book" is intended to help teachers understand what each standard means in terms of what students must know and be able to do. It provides only a *sample* of instructional strategies and examples. The goal of every teacher should be to guide students in understanding & making sense of mathematics.

Construction directions:

Print on cardstock. Cut the tabs on each page starting with page 2. Cut the bottom off of this top cover to reveal the tabs for the subsequent pages. Staple or bind the top of all pages to complete your flip book.

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1. **Make sense of problems and persevere in solving them.**
   In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach.

2. **Reason abstractly and quantitatively.**
   Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and also relate addition and subtraction to length.

3. **Construct viable arguments and critique the reasoning of others.**
   Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask appropriate questions.

4. **Model with mathematics.**
   In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

5. **Use appropriate tools strategically.**
   In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation.

6. **Attend to precision.**
   As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.

7. **Look for and make use of structure. (Deductive Reasoning)**
   Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).

8. **Look for and express regularity in repeated reasoning. (Inductive Reasoning)**
   Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as rounding up and then adjusting the answer to compensate for the rounding. Students continually check their work by asking themselves, "Does this make sense?"

Standards for Mathematical Practice

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. Make sense of problems and persevere in solving them. • Interpret and make meaning of the problem to find a starting point. Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.	How would you describe the problem in your own words? How would you describe what you are trying to find? What do you notice about...? What information is given in the problem? Describe the relationship between the quantities. Describe what you have already tried. What might you change? Talk me through the steps you've used to this point. What steps in the process are you most confident about? What are some other strategies you might try? What are some other problems that are similar to this one? How might you use one of your previous problems to help you begin? How else might you organize...represent... show...?
2. Reason abstractly and quantitatively. • Make sense of quantities and their relationships. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	What do the numbers used in the problem represent? What is the relationship of the quantities? How is _____ related to _____? What is the relationship between _____and _____? What does_____mean to you? (e.g. symbol, quantity, diagram) What properties might we use to find a solution? How did you decide in this task that you needed to use...? Could we have used another operation or property to solve this task? Why or why not?
3. Construct viable arguments and critique the reasoning of others. • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	What mathematical evidence would support your solution? How can we be sure that...? / How could you prove that...? Will it still work if...? What were you considering when...? How did you decide to try that strategy? How did you test whether your approach worked? How did you decide what the problem was asking you to find? (What was unknown?) Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? What is the same and what is different about...? How could you demonstrate a counter-example?
4. Model with mathematics. • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the mathematics they know to solve everyday problems. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving/revising the model. • Ask themselves, "How can I represent this mathematically?"	What number model could you construct to represent the problem? What are some ways to represent the quantities? What is an equation or expression that matches the diagram, number line.., chart..., table..? Where did you see one of the quantities in the task in your equation or expression? How would it help to create a diagram, graph, table...? What are some ways to visually represent...? What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. Use appropriate tools strategically. • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	What mathematical tools could we use to visualize and represent the situation? What information do you have? What do you know that is not stated in the problem? What approach are you considering trying first? What estimate did you make for the solution? In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? What can using a _____ show us that _____ may not? In what situations might it be more informative or helpful to use...?
6. Attend to precision. • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	What mathematical terms apply in this situation? How did you know your solution was reasonable? Explain how you might show that your solution answers the problem. What would be a more efficient strategy? How are you showing the meaning of the quantities? What symbols or mathematical notations are important in this problem? What mathematical language...,definitions..., properties can you use to explain...? How could you test your solution to see if it answers the problem?
7. Look for and make use of structure. • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	What observations do you make about...? What do you notice when...? What parts of the problem might you eliminate..., simplify...? What patterns do you find in...? How do you know if something is a pattern? What ideas that we have learned before were useful in solving this problem? What are some other problems that are similar to this one? How does this relate to...? In what ways does this problem connect to other mathematical concepts?
8. Look for and express regularity in repeated reasoning. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results	Explain how this strategy work in other situations? Is this always true, sometimes true or never true? How would we prove that...? What do you notice about...? What is happening in this situation? What would happen if...? Is there a mathematical rule for...? What predictions or generalizations can this pattern support? What mathematical consistencies do you notice ?

Critical Areas for Mathematics in 2nd Grade

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Domain: **Operations and Algebraic Thinking (OA)**
Cluster: Represent and solve problems involving addition and subtraction.
Standard: **2.OA.1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Table 1, page 49.)

Standards for Mathematical Practices (MP) to be emphasized:
MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.8. Look for and express regularity in repeated reasoning.

Connections:
This cluster is connected to the Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction**.
This cluster is connected to *Represent and solve problems involving addition and subtraction* and *Work with addition and subtraction equations* in Grade 1, to *Relate addition and subtraction to length* and *Work with time and money* in Grade 2, and to *Solve problems involving the four operations, and identify and explain patterns in arithmetic* in Grade 3.

Explanations and Examples

2.OA.1 calls for students to add and subtract numbers within 100 in the context of one and two step word problems. Students should have ample experiences working on various types of problems that have unknowns in all positions, including:

See CCSS Table 1 page 49 for additional examples as well as subtraction examples:

Results Unknown:	**Change Unknown**:	**Start Unknown**:
There are 29 students on the playground. Then 18 more students showed up. How many students are there now? (29=18=__)	There are 29 students on the playground. Some more students show up. There are now 47 students. How many students came? (29+__=47)	There are some students on the playground. Then 18 more students came. There are now 47 students. How many students were on the playground at the beginning? (__+18=47)

This standard also calls for students to solve one- and two-step problems using drawings, objects and equations. Students can use place value blocks or hundreds charts, or create drawings of place value blocks or number lines to support their work. Examples of one-step problems with unknowns in different places are provided in Table 1, page 49. Two step-problems include situations where students have to add and subtract within the same problem.

Example:
In the morning there are 25 students in the cafeteria. 18 more students come in. After a few minutes, some students leave. If there are 14 students still in the cafeteria, how many students left the cafeteria? Write an equation for your problem. (EXPECT students to use *place value blocks (base 10)*, *number line, hundreds chart*, etc. to show, solve and explain their reasoning.)

Word problems that are connected to students' lives can be used to develop fluency with addition and subtraction. Table 1 describes the four different addition and subtraction situations and their relationship to the position of the unknown.

Examples continue next page

Example:
- Take From (Result unknown): David had 63 stickers. He gave 37 to Susan. How many stickers does David have now? 63 – 37 = ____
- Add To: David had $37. His grandpa gave him some money for his birthday. Now he has $63. How much money did David's grandpa give him? $37 + ___ = $63
- Compare: David has 63 stickers. Susan has 37 stickers. How many more stickers does David have than Susan? 63 – 37 = ____
  - Even though the modeling of the two problems above is different, the equation, 63 - 37 = ___ can represent both situations (How many more do I need to make 63?)
- Take From (Start Unknown): David had some stickers. He gave 37 to Susan. Now he has 26 stickers. How many stickers did David have before? ___ - 37 = 26

It is important to attend to the difficulty level of the problem situations in relation to the position of the unknown.
- Result Unknown, Total Unknown, and Both Addends Unknown problems are the least complex for students.
- The next level of difficulty includes Change Unknown, Addend Unknown, and Difference Unknown
- The most difficult are Start Unknown and versions of Bigger and Smaller Unknown (compare problems).

This standard focuses on developing an algebraic representation of a word problem through addition and subtraction. The intent is NOT to introduce traditional algorithms or rules, but to "**make meaning**" of operations.

Second graders should work on ALL problem types regardless of the level of difficulty. Mastery is expected in second grade. Students can use interactive whiteboard or document camera to demonstrate and justify their thinking.

Instructional Strategies
Solving algebraic problems requires emphasizing the most crucial problem-solving strategy—understand the situation.

Students now build on their work with one-step problems to solve two-step problems and model and represent their solutions with equations for all the situations shown in Table 1 on page 49. The problems should involve sums and differences less than or equal to 100 using the numbers 0 to 100. It is important that students develop the habit of checking their answer to a problem to determine if it makes sense for the situation and the questions being asked.

Ask students to write word problems for their classmates to solve. Start by giving students the answer to a problem. Then tell students whether it is an addition or subtraction problem situation. Also let them know that the sums and differences can be less than or equal to 100 using the numbers 0 to 100. For example, ask students to write an addition word problem for their classmates to solve which requires adding four two-digit numbers with 100 as the answer. Students then share, discuss and compare their solution strategies after they solve the problems.

Common Misconceptions:
Some students end their solution to a two-step problem after they complete the first step. They may have misunderstood the question or only focused on finding the first part of the problem. Students need to check their work to see if their answer makes sense in terms of the problem situation. They need many opportunities to solve a variety of two-step problems and develop the habit of reviewing their solution after they think they have finished.

Many children have misconceptions about the equal sign. Students can misunderstand the use of the equal sign even if they have proficient computational skills. The equal sign means , "is the same as" however, many primary students think that the equal sign tells you that the "answer is coming up." Students need to see examples of number sentences with an operation to the right of the equal sign and the answer on the left, so they do not overgeneralize from those limited examples. They might also be predisposed to think of equality in terms of calculating answers rather than as a relation because it is easier for young children to carry out steps to find an answer than to identify relationships among quantities.

Students might rely on a key word or phrase in a problem to suggest an operation that will lead to an incorrect solution. They might think that the word *left* always means that subtraction must be used to find a solution. Students need to solve problems where key words are contrary to such thinking. For example, the use of the word *left* does not indicate subtraction as a solution method: Debbie took the 8 stickers he no longer wanted and gave them to Anna. Now Debbie has 11 stickers *left*. How many stickers did Debbie have to begin with?

It is important that students avoid using key words to solve problems. The goal is for students to make sense of the problem and understand what it is asking them to do, rather than search for "tricks" and/or guess at the operation needed to solve the problem.

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Add and subtract within 20.

Standard: **2.OA.2**. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. (See standard 1.OA.6 for a list of mental strategies.)

Standards for Mathematical Practices (MP) to be emphasized:
MP.2. Reason abstractly and quantitatively.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:
This cluster is connected to the Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction**.
This cluster is connected to *Represent and solve problems involving addition and subtraction* and *Add and subtract within 20* in Grade 1, and to *Use place value understanding and properties of operations to add and subtract* in Grade 2.

Explanations and Examples:
2.OA2 mentions the word fluently when students are adding and subtracting numbers within 20. Fluency means accuracy (correct answer), efficiency (within 4-5 seconds), and flexibility (using strategies such as making 10 or breaking apart numbers). Research indicates that teachers' can best support students' memorization of sums and differences through varied experiences making 10, breaking numbers apart and working on mental strategies, rather than repetitive timed tests.

Example: 9 + 5= __

Student 1	Student 2
Counting On I started at 9 and then counted 5 more. I landed on 14.	Decomposing a Number Leading to a Ten I know that 9 and 1 is 10, so I broke 5 into 1 and 4. 9 plus 1 is 10. Then I have to add 4 more, which gets me to 14.

Example: 13 - 9 = __

Student 1	Student 2
Using the Relationship between Addition and Subtraction: I know that 9 plus 4 equals 13. So, 13 minus 9 equals 4.	Creating an easier problem using compatible numbers: Instead of 13 minus 9. I added 1 to each of the numbers to make the problem 14 minus 10. I know the answer is 4. So 13 minus 9 is also 4.

This standard is strongly connected to all the standards in this domain. It focuses on students being able to fluently add and subtract numbers to 20. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Mental strategies help students make sense of number relationships as they are adding and subtracting within 20. The ability to calculate mentally with efficiency is very important for all students. Mental strategies may include the following:
- Counting on
- Making tens (9 + 7 = 10 + 6)
- Decomposing a number leading to a ten (14 – 6 = 14 – 4 – 2 = 10 – 2 = 8)
- Fact families (8 + 5 = 13 is the same as 13 - 8 = 5)
- Doubles
- Doubles plus one (7 + 8 = 7 + 7 + 1)

The use of objects, diagrams, or interactive whiteboards, and various strategies will help students develop fluency.
Continued next page

Instructional Strategies:

An efficient strategy is one that can be done mentally and quickly. Provide many activities that will help students develop a strong understanding of number relationships, addition and subtraction so they can develop, share and use efficient strategies for mental computation. Students gain computational fluency, using efficient and accurate methods for computing, as they come to understand the role and meaning of arithmetic operations in number systems. Efficient mental processes become automatic with use.

Have students study how numbers are related to the anchor numbers 5 and 10, so they can apply these relationships to their strategies for knowing 5+4 or 8 +3. Students might picture 5+4 on a ten-frame to mentally see 9 as the answer. For remembering 8+7, students might think, since 8 is 2 away from 10, take 2 away from 7 to make 10+5=15. Another example: After multiple experiences with ten-frames, when students add to 9, they mentally SEE 9, but THINK 10 and generalize that 9+ 8 is the same thing as 10 + 7. Then, apply this same thinking to 19+8 is the same thing as 20 + 7, and so on.

Provide activities in which students apply the commutative and associative properties to their mental strategies for sums less or equal to 20 using the numbers 0 to 20.

Provide simple word problems designed for students to invent and try a particular strategy as they solve it. Have students explain their strategies so their classmates can understand it. Guide the discussion so the focus is on the methods that are most useful. Encourage students to try the strategies that were shared so they can eventually adopt efficient strategies that work for them. Make posters for student-developed mental strategies for addition and subtraction within 20. Use names for the strategies that make sense to the students and include examples of the strategies. Present a particular strategy along with the specific addition and subtraction facts relevant to the strategy. Have students use objects and drawings to explore how these facts are alike.

Common Misconceptions:

Students may overgeneralize the idea that answers to addition problems must be greater. Adding 0 to any number results in a sum that is equal to that number. Provide word problems involving 0 and have students model using drawings with an empty space for 0.

Students are usually proficient when they focus on a strategy relevant to particular facts. When these facts are mixed with others, students may revert to counting as a strategy and ignore the efficient strategies they learned. Provide a list of facts from two or more strategies and ask students to name a strategy that would work for that fact. Students should be expected to explain why they chose that strategy then show how to use it.

2.OA.2

Domain: **Operations and Algebraic Thinking (OA)**
Cluster: Work with equal groups of objects to gain foundations for multiplication.
Standard: **2.OA.3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
Standards for Mathematical Practices (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP.3, Construct viable arguments and critique the reasoning of others. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Connections: **(2.OA.3-4)** This cluster is connected to the Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction** and #4, **Describing and analyzing shapes**. This cluster is connected to *Work with addition and subtraction equations* and *Use place value understanding and properties of operations to add and subtract* in Grade 1, and to *Use place value understanding and properties of operations to add and subtract* in Grade 2.
Explanations and Examples: 2.OA.3 calls for students to apply their work with doubles addition facts to the concept of odd or even numbers. Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends (e.g., 10 = 5 +5), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, place value cubes, etc.) before moving towards pictorial representations such as circles or arrays. **Example**: Is 8 an even number? Prove your answer. Student 1 I grabbed 8 counters. I paired counters up into groups of 2. Since I didn't have any counters left over, I know that 8 is an even number. Student 2 Creating an Easier Problem: Instead of 13 minus 9, I added 1 to each of the numbers to make the problem 14 minus 10. I know the answer is 4. So 13 minus 9 is also 4. Student 3 I drew 8 boxes in a rectangle that had two columns. Since every box on the left matches a box on the right, I know that 8 is even. Student 4 I drew 8 circles. I matched one on the left with one on the right. Since they all match up I know that 8 is an even number. Student 5 I know that 4 plus 4 equals 8. So 8 is an even number.   Continued next page

Students explore odd and even numbers in a variety of ways including the following: students may investigate if a number is odd or even by determining if the number of objects can be divided into two equal sets, arranged into pairs or counted by twos. After the above experiences, students may discover that they only need to look at the digit in the ones place to determine if a number is odd or even since any number of tens will always split into two even groups.
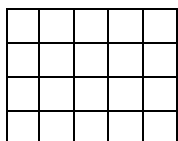
Example:
Students need opportunities writing equations representing sums of two equal addends, such as: 2 + 2 = 4, 3 + 3 = 6, 5 + 5 = 10, 6 + 6 = 12, or 8 + 8 =16. This understanding will lay the foundation for multiplication and is closely connected to 2.OA.4.

The use of objects and/or interactive whiteboards will help students develop and demonstrate various strategies to determine even and odd numbers.

Instructional Strategies: (2.OA.3-4)

Students need to understand that a collection of objects can be one thing or one group and that a group contains a given number of objects. Investigate separating no more than 20 objects into two equal groups. Find the numbers that will have some objects remaining and no objects remaining after separating the collections into two equal groups. Odd numbers will have some objects remaining while even numbers will not. For an even number of objects in a collection, show the total as the sum of equal addends (repeated addition).

A rectangular array is an arrangement of objects in horizontal rows and vertical columns. Arrays can be made out of any number of objects that can be put into rows and columns. All rows contain the same number of items and all columns contain an equal number of items. Have students use objects to build all the arrays possible with no more than 25 objects. Their arrays should have up to 5 rows and up to 5 columns. Ask students to draw the arrays on grid paper and write two different equations under the arrays: one showing the total as a sum by rows and the other showing the total as a sum by columns. Both equations will show the total as a sum of equal addends.

The equation by rows: 20 = 5 + 5 + 5 +5

The equation by columns: 20 = 4 + 4 + 4 + 4 + 4

Build on knowledge of composing and decomposing numbers to investigate arrays with up to 5 rows and up to 5 columns in different orientations. For example, form an array with 3 rows and 4 objects in each row. Represent the total number of objects with equations showing a sum of equal addends two different ways: by rows, 12 = 4 + 4 + 4; by columns, 12 = 3 + 3 + 3 + 3. Show that by rotating the array 90° to form 4 rows with 3 objects in each row. Write two different equations to represent 12 as a sum of equal addends: by rows, 12 = 3 + 3 + 3 + 3; by columns, 12 = 4 + 4 + 4. Have students discuss this statement and explain their reasoning: The two arrays are different and yet the same.

Ask students to think of a full ten-frame showing 10 circles as an array. One view of the ten-frame is 5 rows with 2 circles in each row. Students count by rows to 10 and write the equation 10 = 2 + 2 + 2 + 2 + 2. Then students put two full ten-frames together end-to-end so they form 10 rows of 2 circles or (10 columns of 2 circles). They use this larger array to count by 2s up to 20 and write an equation that shows 20 equal to the sum of ten 2s.

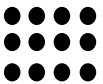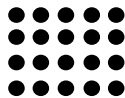| Domain: **Operations and Algebraic Thinking (OA)** |
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| Cluster: Work with equal groups of objects to gain foundations for multiplication. |
| Standard: **2.OA.4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. |
| **Standards for Mathematical Practices (MP) to be emphasized:**
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.7. Look for and make use of structure.
 MP.8. Look for and express regularity in repeated reasoning. |
| **Connections**:
See 2.OA.3 |
| **Explanations and Examples:**
2.OA.4 calls for students to use rectangular arrays to work with repeated addition. This is a building block for multiplication in 3rd Grade. Students should explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Based on the commutative property of addition, students can add either the rows or the columns and still arrive at the same solution.

Example below:
Find the total number of objects below. (Students use concrete objects to show, solve and explain)

Student 1
I see 3 counters in each column and there are 4 columns. So I added
3 + 3 + 3 + 3. That equals 12.

Student 2
I see 4 counters in each row and there are 3 rows. So I added 4+4+4. That equals 12.

Students may arrange any set of objects into a rectangular array. Objects can be cubes, buttons, counters, etc. Objects do not have to be square to make an array. *Geoboards* can also be used to demonstrate rectangular arrays. Students then write equations that represent the total as the sum of equal addends as shown below.

4 + 4 + 4 =   5 + 5 + 5 + 5 = 20

Interactive whiteboards and document cameras may be used to help students visualize and create arrays. |

Second Grade Operations and Algebraic Thinking
Extended Common Core State Standards Mathematics

The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "...materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication. North Carolina DOE

Second Grade Mathematics
Operations and Algebraic Thinking (OA)

Common Core State Standards	Essence	Extended Common Core
Represent and solve problems involving addition and subtraction	**Use addition and subtraction to solve problems.**	**Represent and solve problems involving addition and subtraction (0-30).**
Cluster 1. Use addition and subtraction within 100 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.		**Cluster** 1. Use objects and representations to add and subtract groups of objects. 2. Use objects, representations and numerals to add and subtract within real life one-step story problems to at least 30.
Work with equal groups of objects to gain foundations for multiplication.	Share fairly to create equal groups.	**Work with equal groups of objects to gain foundations for multiplication.**
2.Determine whether a group of objects (up to 20)has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. 3. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.		3. Share fairly collections of up to 20 items between 2-4 people. 4. Describe set as "same quantity" after breaking apart and reassembling a given quantity (up to ten). 5. 5. Determine whether two or more groups of objects (up to 20) has an odd or even number of members, e.g., by pairing objects; determine equivalent relationships, using the equal symbol (=).

**EXTENDED
2.OA**

Cluster: Understand place value.

Standard: **2.NBT.1**. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens—called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Standards for Mathematical Practices (MP) to be emphasized:
MP.2. Reason abstractly and quantitatively.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:
This cluster is connected to the Second Grade Critical Area of Focus #1, **Extending understanding of base-ten notation**.
This cluster is connected to *Extend the counting sequence* and *Understand place value* in Grade 1, to *Work with equal groups of objects to gain foundations for multiplication* in Grade 2, and to *Use place value understanding and properties of operations to perform multi-digit arithmetic* in Grade 3.

Explanations and Examples: (2.NBT.1–4)
2.NBT.1 calls for students to work on decomposing numbers by place value. Students should have ample experiences with concrete materials and pictorial representations examining that numbers all numbers between 100 and 999 can be decomposed into hundreds, tens, and ones. Interpret the value of a digit (1-9 and 0) in a multi-digit numeral by its position within the number with models, words and numerals.
Use 10 as a benchmark number to compose and decompose when adding and subtracting whole numbers.
   2.NBT.1a calls for students to extend their work from 1st Grade by exploring a hundred as a unit (or bundle) of ten tens.
   2.NBT.1b builds on the work of **2.NBT.2a.** Students should explore the idea that numbers such as 100, 200, 300, etc., are groups of hundreds that have no tens or ones. Students can represent this with place value (base 10) blocks.
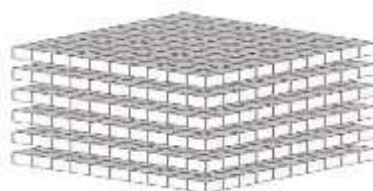
is the same as

6 hundreds are the same as 600

Understanding that 10 ones make one ten and that 10 tens make one hundred is fundamental to students' mathematical development. Students need multiple opportunities counting and "bundling" groups of tens in first grade. In second grade, students build on their understanding by making bundles of 100s with or without leftovers using base ten blocks, cubes in towers of 10, ten frames, etc. This emphasis on bundling hundreds will support students' discovery of place value patterns.

As students are representing the various amounts, it is important that emphasis is placed on the language associated with the quantity. For example, 243 **can be expressed in multiple ways** such as 2 groups of hundred, 4 groups of ten and 3 ones, as well as 24 tens and 3 ones. When students read numbers, they should read in standard form as well as using place value concepts. For example, 243 should be read as "two hundred forty-three" as well as two hundreds, 4 tens, 3 ones. (A document camera or interactive whiteboard can also be used to demonstrate "bundling" of objects. This gives students the opportunity to communicate their thinking).

Instructional Strategies: (2.NBT.1-4)

The understanding that 100 is 10 tens or 100 ones is critical to the understanding of place value. Using proportional models like base-ten blocks and bundles of tens along with numerals on place-value mats provides connections between physical and symbolic representations of a number. These models can be used to compare two numbers and identify the value of their digits.

Model three-digit numbers using base-ten blocks in multiple ways. For example, 236 can be 236 ones, or 23 tens and 6 ones, or 2 hundreds, 3 tens and 6 ones, or 20 tens and 36 ones. Use activities and games that have students match different representations of the same number.

Provide games and other situations that allow students to practice skip-counting. Students can use nickels, dimes and dollar bills to skip count by 5, 10 and 100. Pictures of the coins and bills can be attached to models familiar to students: a nickel on a five-frame with 5 dots or pennies and a dime on a ten-frame with 10 dots or pennies.

On a number line, have students use a clothespin or marker to identify the number that is ten more than a given number or five more than a given number.

Have students create and compare all the three-digit numbers that can be made using numbers from 0 to 9. For instance, using the numbers 1, 3, and 9, students will write the numbers 139, 193, 319, 391, 913 and 931. When students compare the numerals in the hundreds place, they should conclude that the two numbers with 9 hundreds would be greater than the numbers showing 1 hundred or 3 hundreds. When two numbers have the same digit in the hundreds place, students need to compare their digits in the tens place to determine which number is larger.

Common Misconceptions: (2.NBT.1-4)

Some students may not move beyond thinking of the number 358 as 300 ones plus 50 ones plus 8 ones to the concept of 8 singles, 5 bundles of 10 singles or tens, and 3 bundles of 10 tens or hundreds. Use base-ten blocks to model the collecting of 10 ones (singles) to make a ten (a rod) or 10 tens to make a hundred (a flat). It is important that students connect a group of 10 ones with the word *ten* and a group of 10 tens with the word *hundred.*

2.NBT.1

Domain: **Number and Operations in Base Ten (NBT)**
Cluster: Understand place value.
Standard: **2.NBT.2.** Count within 1000; skip-count by 5s, 10s, and 100s.

Standards for Mathematical Practices (MP) to be emphasized:
MP.2. Reason abstractly and quantitatively.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:
See 2.NBT.1

Explanations and Examples:
2.NBT.2 calls for students to count within 1,000. This means that students are expected to "count on" from any number and say the next few numbers that come afterwards.

Understand that counting by 2s, 5s and 10s is counting groups of items by that amount.

Example:
What are the next 3 numbers after 498? *499, 500, 501*.
When you count back from 201, what are the first 3 numbers that you say? *200, 199, 198*.
This standard also introduces skip counting by 5s and 100s. Students are introduced to skip counting by 10s in First Grade. Students should explore the patterns of numbers when they skip count. When students skip count by 5s, the ones digit alternates between 5 and 0. When students skip count by 100s, the hundreds digit is the only digit that changes, and it increases by one number.

Students need many opportunities counting, up to 1000, from different starting points. They should also have many experiences skip counting by 5s, 10s, and 100s to develop the concept of place value.

Examples:
- The use of the 100s chart may be helpful for students to identify the counting patterns.
- The use of money (nickels, dimes, dollars) or base ten blocks may be helpful visual cues.
- The use of an interactive whiteboard may also be used to develop counting skills.

The ultimate goal for second graders is to be able to count in multiple ways with no visual support.

Domain: **Number and Operations in Base Ten (NBT)**
Cluster: Understand place value.
Standard: **2.NBT.3**. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
Standards for Mathematical Practices (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP. 6 Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Connections: See 2.NBT.1
Explanations and Examples: **2.NBT.3** calls for students to read, write and represent a number of objects with a written numeral (number form or standard form). These representations can include place value (base 10) blocks, pictorial representations or other concrete materials. Remember that when reading and writing whole numbers, the word "and" should **not** be used. **Example**: 235 is written as two hundred thirty-five. Students need many opportunities reading and writing numerals in multiple ways. **Examples**:   - Base-ten numerals   637   (standard form)   - Number names   six hundred thirty seven   (written form)   - Expanded form   600 + 30 + 7   (expanded notation)   When students say the expanded form, it may sound like this: "6 hundreds plus 3 tens plus 7 ones" OR 600 plus 30 plus 7."

Domain: **Number and Operations in Base Ten (NBT)**
Cluster: Understand place value.
Standard: **2.NBT.4**. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
Standards for Mathematical Practices (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Connections: See 2.NBT.1
Explanations and Examples: **2.NBT.4** builds on the work of **2.NBT.1** and **2.NBT.3** by having students compare two numbers by examining the amount of hundreds, tens and ones in each number. Students are introduced to the symbols greater than (>), less than (<) and equal to (=) in First Grade, and use them in Second Grade with numbers within 1,000. Students should have ample experiences communicating their comparisons in words before using only symbols in this standard. Example: 452 ___ 455 Student 1 452 has 4 hundreds 5 tens and 2 ones. 455 has 4 hundreds 5 tens and 5 ones. They have the same number of hundred and the same number of tens, but 455 has 5 ones and 452 only has 2   ones. 452 is less than 455. 452<455 Student 2 452 is less than 455. I know this because when I count up I say 452 before I say 455. Students may use models, number lines, base ten blocks, interactive whiteboards, document cameras, written words, and/or spoken words that represent two three-digit numbers. To compare, students apply their understanding of place value. They first attend to the numeral in the hundreds place, then the numeral in tens place, then, if necessary, to the numeral in the ones place. Comparative language includes but is not limited to: more than, less than, greater than, most, greatest, least, same as, equal to and not equal to. Students use the appropriate symbols to record the comparisons.

Domain: **Number and Operations in Base Ten (NBT)**
Cluster: Use place value understanding and properties of operations to add and subtract.
Standard: **2.NBT.5.** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Standards for Mathematical Practices (MP) to be emphasized:
MP.2. Reason abstractly and quantitatively.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections: **(2.NBT.5-9)**
This cluster is connected to the Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction**.
This cluster is connected to *Understand and apply properties of operations and the relationship between addition and subtraction* and *Add and subtract within 20 and Use place value understanding and properties of operations to add and subtract* in Grade 1, to *Add and subtract within 20* in Grade 2, and to *Use place value understanding and properties of operations to perform multi-digit arithmetic* in Grade 3.

Explanations and Examples:
2.NBT.5 mentions the word fluently when students are adding and subtracting numbers within 100. Fluency means accuracy (correct answer), efficiency (reasonable steps and time in computing), and flexibility (using strategies such as making 10 or breaking numbers apart).

This standard calls for students to use pictorial representations or strategies to find the solution. Students who are struggling may benefit from further work with concrete objects (e.g., place value blocks).

Example: 67 + 25 = __

Place Value Strategy:	Counting On and Decomposing a Number Leading to a Ten:	Commutative Property:
I broke both 67 and 25 into tens and ones. 6 tens plus 2 tens equals 8 tens. Then I added the ones. 7 ones plus 5 ones equals 12 ones. I then combined my tens and ones. 8 tens plus 12 ones	I wanted to start with 67 and then break 25 apart. I started with 67 and counted on to my next ten. 67 plus 3 gets me to 70. I then added 2 more to get to 72. I then added my 20 and got to 92.	I broke 67 and 25 into tens and ones so I had to add 60+7+20+5. I added 60 and 20 first to get 80. Then I added 7 to get 87. Then I added 5 more. My answer is 92.

Example: 63 – 32 = __

Relationship between Addition and Subtraction:
I broke apart both 63 and 32 into tens and ones. I know that 2 plus 1 equals 3, so I have 1 left in the ones place. I know that 3 plus 3 equals 6, so I have a 3 in my tens place. My answer has a 1 in the ones place and 3 in the tens place, so my answer is 31.

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Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Students should have experiences solving problems written both horizontally and vertically. They need to communicate their thinking and be able to justify their strategies both verbally and with paper and pencil.

Addition strategies based on place value for 48 + 37 may include:
- Adding by place value: 40 + 30 = 70 and 8 + 7 = 15 and 70 + 15 = 85.
- Incremental adding (breaking one number into tens and ones); 48 + 10 = 58, 58 + 10 = 68, 68 + 10 = 78, 78 + 7 = 85
- Compensation (making a friendly number): 48 + 2 = 50, 37 – 2 = 35, 50 + 35 = 85

Subtraction strategies based on place value for 81 - 37 may include:
- Adding up (from smaller number to larger number): 37 + 3 = 40, 40 + 40 = 80, 80 + 1 = 81, and 3 + 40 + 1 = 44.
- Incremental subtracting: 81 -10 = 71, 71 – 10 = 61, 61 – 10 = 51, 51 – 7 = 44
- Subtracting by place value: 81 – 30 = 51, 51 – 7 = 44

Properties that students should know and use are:
- Commutative property of addition (Example: 3 + 5 = 5 + 3)
- Associative property of addition (Example: (2 + 7) + 3 = 2 + (7+3))
- Identity property of 0 (Example: 8 + 0 = 8)

Students in second grade need to communicate their understanding of why some properties work for some operations and not for others.
- Commutative Property: In first grade, students investigated whether the commutative property works with subtraction. The intent was for students to recognize that taking 5 from 8 is not the same as taking 8 from 5. Students should also understand that they will be working with numbers in later grades that will allow them to subtract greater numbers from smaller numbers. This exploration of the commutative property continues in second grade.

Associative Property: It is difficult for students to recognize that the associative property does not work for subtraction at this grade level because it is a challenge to determine all the possibilities.

Common Misconceptions: (2.NBT.5-9)
Students may think that the 4 in 46 represents 4, not 40. Students need many experiences representing two-and three-digit numbers with manipulatives that group (base ten blocks) and those that do NOT group, such as counters, etc.

When adding two-digit numbers, some students might start with the digits in the ones place and record the entire sum. Then they add the digits in the tens place and record this sum. Assess students' understanding of *a ten* and provide more experiences modeling addition with grouped and pregrouped base-ten materials as mentioned above.

When subtracting two-digit numbers, students might start with the digits in the ones place and subtract the smaller digit from the greater digit. Then they move to the tens and the hundreds places and subtract the smaller digits from the greater digits. Assess students' understanding of *a ten* and provide more experiences modeling subtraction with grouped and pregrouped base-ten materials.

Domain: **Number and Operations in Base Ten (NBT)**

Cluster: Use place value understanding and properties of operations to add and subtract.

Standard: **2.NBT.6**. Add up to four two-digit numbers using strategies based on place value and properties of operations.

Standards for Mathematical Practices (MP) to be emphasized:
MP.2. Reason abstractly and quantitatively.
MP.7. Look for and make use of structure.
MP.6 Attend to precision.
MP.8. Look for and express regularity in repeated reasoning.

Connections:
See 2.NBT.5

Explanations and Examples:
2.NBT.6 calls for students to add a string of two-digit numbers (up to four numbers) by applying place value strategies and properties of operations.

Example:
43 + 34 + 57 + 24 = ___

Student 1
Associative Property
I saw the 43 and 57 and added them first, since I know 3 plus 7 equals 10. When I added them 100 was my answer. Then I added 34 and had 134. Then I added 24 and had 158.

Student 2
Place Value Strategies
I broke up all of the numbers into tens and ones. First I added the tens. 40 + 30 + 50 + 20 = 140.
Then I added the ones. 3 + 4 + 7 + 4 = 18. Then I combined the tens and ones and had 158 as my answer.

Student 3
Place Value Strategies and Associative Property
I broke up all the numbers into tens and ones. First I added up the tens.
40 + 30 + 50 + 20. I changed the order of the numbers to make adding easier. I know that 30 plus 20 equals 50 and 50 more equals 100. Then I added the 40 and got 140.

Student 4
Then I added up the ones. 3 + 4 + 7 + 4. I changed the order of the numbers to make adding easier. I know that 3 plus 7 equals 10 and 4 plus 4 equals 8. 10 plus 8 equals 18.
I then combined my tens and my ones. 140 plus 18 equals 158.

Students demonstrate addition strategies with up to four two-digit numbers either with or without regrouping. Problems may be written in a story problem format to help develop a stronger understanding of larger numbers and their values. Interactive whiteboards and document cameras may also be used to model and justify student thinking.

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2.NBT.6

Domain: **Number and Operations in Base Ten (NBT)**
Cluster: Use place value understanding and properties of operations to add and subtract.
Standard: **2.NBT.7**. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
Standards for Mathematical Practices (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Connections: See 2.NBT.5
Explanations and Examples: **2.NBT.7** builds on the work from 2.NBT.5 by increasing the size of numbers (two 3-digit numbers). Students should have ample experiences to use concrete materials (place value blocks) and pictorial representations to support their work. This standard also references composing and decomposing a ten. This work should include strategies such as making a 10, making a 100, breaking apart a 10, or creating an easier problem. While the standard (traditiona)l algorithm could be used here, students' experiences should extend beyond only working with the algorithm. There is a strong connection between this standard and place value understanding with addition and subtraction of smaller numbers. Students may use concrete models or drawings to support their addition or subtraction of larger numbers. Strategies are similar to those stated in 2.NBT.5, as students extend their learning to include greater place values moving from tens to hundreds to thousands. Interactive whiteboards and document cameras may also be used to model and justify student thinking. Students use number lines, base ten blocks, etc. to show, solve and explain reasoning. **Example: 354 287 = __** Student 1 uses a number line. "I started at 354 and jumped 200. I landed on 554. I then made 8 jumps of 10 and landed on 634. I then jumped 7 and landed on 641." Student 2 uses base ten blocks & mat. "I broke all of the numbers up by place using a place value chart. I first added the ones(4+7), then the tens (50+80)and then the hundreds200=500) I then combined my answers: 500+130=630. 630+11=641". Student 3 uses place value blocks. "I made a pile of 354. I then added 287. That gave me 5 hundreds, 13 tens and 11 ones. I noticed that I could trade some pieces. I had 11 ones, and traded 10 ones for a ten. I then had 14 tens, so I traded 10 tens for a hundred I ended up with 6 hundreds, 4 tens and 11 ones.

Domain: **Number and Operations in Base Ten (NBT)**
Cluster: Use place value understanding and properties of operations to add and subtract.
Standard: **2.NBT.8.** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
Standards for Mathematical Practices (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Connections: See 2.NBT.5
Explanations and Examples: **2.NBT.8** calls for students to mentally add or subtract multiples of 10 or 100 to any number between 100 and 900. Students should have ample experiences working with the concept of adding and subtracting multiples of 10 or 100 that you are only changing the tens place (multiples of ten) or the digit in the hundreds place (multiples of 100). In this standard, problems that require students to move from 10's to 100's should be included. **Example: 273 + 60 = 333.** Students need many opportunities to practice mental math by adding and subtracting multiples of 10 and 100 up to 900 using different starting points. They can practice this by counting and thinking aloud, finding missing numbers in a sequence, and finding missing numbers on a number line or hundreds chart. Explorations should also include looking for relevant patterns. Mental math strategies may include: • counting on; 300, 400, 500, etc. • counting back; 550, 450, 350, etc. Examples: • 100 more than 653 is _____ (753) • 10 less than 87 is _____ (77) • "Start at 248. Count up by 10s until I tell you to stop." An interactive whiteboard or document camera may be used to help students develop these mental math skills. Continued next page

Domain: **Number and Operations in Base Ten (NBT)**
Cluster: Use place value understanding and properties of operations to add and subtract.
Standard: **2.NBT.9**. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)
Standards for Mathematical Practices (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Explanations and Examples:

2.NBT.9 calls for students to explain using concrete objects, pictures and words (oral or written) **why** addition or subtraction strategies work. The expectation is that students apply their knowledge of place value and the properties of operations in their explanation.

Students should have the opportunity to solve problems and then explain why their strategies work.

Example:
There are 36 birds in the park. 25 more birds arrive. How many birds are there? Solve the problem and show your work.

Student 1
I broke 36 and 25 into tens and ones and then added them. 30 + 6 + 20 + 5. I can change the
order of my numbers, so I added 30+20 and got 50. Then I added on 6 to get 56. Then I added
5 to get 61. This strategy works because I broke all the numbers up by their place value.

Student 2
I used place value blocks and made a pile of 36. Then I added 25. I had 5 tens and 11 ones.
I had to trade 10 ones for a 10. Then I had 6 tens and 1 one. That makes 61. This strategy
works because I added up the tens and then added up the ones and traded if I had more than
10 ones.

Students could also have experiences examining strategies and explaining why they work. Also include incorrect examples for students to examine. Operations embedded within meaningful context promote development of reasoning and justification.

Example:
One of your classmates solved the problem 56 - 34 = ___ by writing ─I know that I need to add 2 to the number 4 to get 6. I also know that I need to add 20 to 30 to get 20 to get to 50. So, the answer is 22.‖ Is their strategy correct? Explain why or why not?

Example:
One of your classmates solved the problem 25 + 35 by adding 20 + 30 + 5 + 5. Is their strategy correct? Explain why or why not?

Example:
Mason read 473 pages in June. He read 227 pages in July. How many pages did Mason read altogether?

- Karla's explanation: 473 + 227 = _____. I added the ones together (3 + 7) and got 10. Then I added the tens together (70 + 20) and got 90. I knew that 400 + 200 was 600. So I added 10 + 90 for 100 and added 100 + 600 and found out that Mason had read 700 pages altogether.

- Debbie's explanation: 473 + 227 = _____. I started by adding 200 to 473 and got 673. Then I added 20 to 673 and I got 693 and finally I added 7 to 693 and I knew that Mason had read 700 pages altogether.

- Becky's explanation: I used base ten blocks on a base ten mat to help me solve this problem. I added 3 ones (units) plus 7 ones and got 10 ones which made one ten. I moved the 1 ten to the tens place. I then added 7 tens rods plus 2 tens rods plus 1 tens rod and got 10 tens or 100. I moved the 1 hundred to the hundreds place. Then I added 4 hundreds plus 2 hundreds plus 1 hundred and got 7 hundreds or 700. So Mason read 700 books.

Students should be able to connect different representations and explain the connections. Representations can include numbers, words (including mathematical language), pictures, number lines, and/or physical objects. Students should be able to use any/all of these representations as needed.

An interactive whiteboard or document camera can be used to help students develop and explain their thinking.

2.NBT.9

The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "...materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication. (North Carolina DOE)

Second Grade Mathematics
Number and Operations in Base Ten (NBT)

	Common Core State Standards	Essence		Extended Common Core
	Understand place value	**Understand place value in base ten**		**Understand place value.**

Cluster	1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
	  a. 100 can be thought of as a bundle of ten tens — called a "hundred."
	  b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
	2. Count within 1000; skip-count by 5s, 10s, and 100s.
	3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
	4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
				1. Count (0-30) by indicating one object at a time (one-to one tagging) using one counting word for every object (synchrony), while keeping track of objects that have and have not been counted.
				2. Write or use an alternative pencil to write numbers 0-30.
				3. Use a number line (0-30) to determine the number before and after (1 more and 1 less).
				4. Use number word (0-30) of last object counted in a set, to name the total number of objects in the set when asked, "How many?" (Cardinality).
				5. Illustrate whole numbers to 30 using objects, representations and numbers.
				6. Compare sets of objects and numbers using appropriate vocabulary (more, less, equal, one more, one less, etc.).
				7. Determine how many more to ten.

	Use place value understanding and properties of operations to add and subtract	Add & subtraction	**Cluster**	**Use place value understanding to add and subtract**
	5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. 6. Add up to four two-digit numbers using strategies based on place value and properties of operations. 7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. 8. Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100- 900. 9. Explain why addition and subtraction strategies work, using place value and the properties of operations.			8. Use part-part-whole relationships (including 2 or more parts) to compose and decompose numbers. 9. Compare numbers (0-30) in relationship to benchmark number 10. 10. Use objects, representations and numbers (0-30) to add and subtract. 11. Use objects and representations (0-30) to add and subtract groups using real life story problems.

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Domain: **Measurement and Data (MD)**

Cluster: Measure and estimate lengths in standard units.

Standard: **2.MD.1.** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Standards for Mathematical Practices (MP) to be emphasized:
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

Connections:
This cluster is connected to the Second Grade Critical Area of Focus #3, **Using standard units of measure**.

This cluster connects to *Measure lengths indirectly and by iterating length units* in Grade 1, and to *Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures* in Grade 3.

Explanations and Examples:
2.MD.1 calls for students to measure the length of objects in both customary (inches and feet) and metric (centimeters and meters). Students should have ample experiences choosing objects, identifying the appropriate tool and unit, and then measuring the object. The teacher should allow students to determine which tools and units to use.

Foundational understandings to help with measure concepts:
- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).
- Understand the relationship between the size of a unit and the number of units needed (compensatory principal).
- Understand the measuring of two-dimensional space (area) using non-standard units.

Students in second grade will build upon what they learned in first grade from measuring length with non-standard units to the new skill of measuring length in metric and U.S. Customary with standard units of measure. They should have many experiences measuring the length of objects with rulers, yardsticks, meter sticks, and tape measures. They will need to be taught how to actually use a ruler appropriately to measure the length of an object especially as to where to begin the measuring. It is important to help students locate the starting point on the measuring instrument, especially when some have a protected edge. Asking students such as: "Do you start at the end of the ruler or at the zero?" helps them focus on where to start on the instrument.

Continued next page

Instructional Strategies (2.MD.1-4)

Second graders are transitioning from measuring lengths with informal or nonstandard units to measuring with these standard units: inches, feet, centimeters, and meters. The measure of length is a count of how many units are needed to match the length of the object or distance being measured. Students have to understand what a length unit is and how it is used to find a measurement. They need many experiences measuring lengths with appropriate tools so they can become very familiar with the standard units and estimate lengths. Use language that reflects the approximate nature of measurement, such as the length of the room is about 26 feet.

Have students measure the same length with different-sized units then discuss what they noticed. Ask questions to guide the discussion so students will see the relationship between the size of the units and measurement, i.e. the measurement made with the smaller unit is more than the measurement made with the larger unit and vice versa.

Insist that students always estimate lengths before they measure. Estimation helps them focus on the attribute to be measured, the length units, and the process. After they find measurements, have students discuss the estimates, their procedures for finding the measurements and the differences between their estimates and the measurements.

Instructional Resources/Tools
Centimeter rulers and tapes
Inch rulers and tapes
Yardsticks
Meter sticks

Common Misconceptions: (2.MD.1-4)

When some students see standard rulers with numbers on the markings, they believe that the numbers are counting the marks instead of the units or spaces between the marks. Have students use informal or standard length units to make their own rulers by marking each whole unit with a number in the middle. They will see that the ruler is a representation of a row of units and focus on the spaces.

Some students might think that they can only measure lengths with a ruler starting at the left edge. Provide situations where the ruler does not start at zero. For example, a ruler is broken and the first inch number that can be seen is 2. If a pencil is measured and it is 9 inches on this ruler, the students must subtract 2 inches from the 9 inches to adjust for where the measurement started.

Domain: **Measurement and Data (MD)**
Cluster: Measure and estimate lengths in standard units.
Standard: **2.MD.2.** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
Standards for Mathematical Practices (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure.
Connections: See 2.MD.1
Explanations and Examples: **2.MD.2** calls for students to measure an object using two units of different lengths. **Example**: A student measures the length of their desk and finds that it is 3 feet and 36 inches. Students should explore the idea that the length of the desk is larger in inches than in feet, since inches are smaller units than feet. This concept is referred to as the compensatory principle. Note: this standard does not specify whether the units have to be within the same system. Students need multiple opportunities to measure using different units of measure. They should not be limited to measuring within the same standard unit. Students should have access to tools, both U.S. Customary and metric. The more students work with a specific unit of measure, the better they become at choosing the appropriate tool when measuring. Students measure the length of the same object using different tools (ruler with inches, ruler with centimeters, a yardstick, or meter stick). This will help students learn which tool is more appropriate for measuring a given object. They describe the relationship between the size of the measurement unit and the number of units needed to measure something. For instance, a student might say, "The longer the unit, the fewer I need." Multiple opportunities to explore provide the foundation for relating metric units to customary units, as well as relating within customary (inches to feet to yards) and within metric (centimeters to meters).

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2.MD.2

Domain: **Measurement and Data (MD)**
Cluster: Measure and estimate lengths in standard units.
Standard: **2.MD.3**. Estimate lengths using units of inches, feet, centimeters, and meters.
Standards for Mathematical Practices (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP.5. Use appropriate tools strategically. MP.6. Attend to precision.
Connections: See 2.MD.1

Explanations and Examples:
2.MD.3 calls for students to estimate the lengths of objects using inches, feet, centimeters, and meters. Students should make estimates after seeing a benchmark unit, such as the length of one inch, before making their estimate.

Example:
Look at your ruler to see how long one inch is. Now, estimate the length of this paper in inches.

Estimation helps develop familiarity with the specific unit of measure being used. To measure the length of a shoe, knowledge of an inch or a centimeter is important so that one can approximate the length in inches or centimeters. Students should begin practicing estimation with items which are familiar to them (length of desk, pencil, favorite book, etc.).

Some useful benchmarks for measurement are: (these are for adults so they will need to be adjusted for 2nd graders)
- First joint to the tip of a thumb is about an inch
- Length from your elbow to your wrist is about a foot
- If your arm is held out perpendicular to your body, the length from your nose to the tip of your fingers is about a yard

Domain: **Measurement and Data (MD)**
Cluster: Measure and estimate lengths in standard units.
Standard: **2.MD.4.** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
Standards for Mathematical Practices (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP.5. Use appropriate tools strategically. MP.6. Attend to precision.
Connections: See 2.MD.1
Explanations and Examples: **2.MD.4** calls for students to determine the difference in length between two objects. Students should choose objects, identify appropriate tools and units, measure both objects, and then determine the differences in lengths. Second graders should be familiar enough with inches, feet, yards, centimeters, and meters to be able to compare the differences in lengths of two objects. They can make direct comparisons by measuring the difference in length between two objects by laying them side by side and selecting an appropriate standard length unit of measure. Students should use comparative phrases such as "It is longer by 2 inches" or "It is shorter by 5 centimeters" to describe the difference between two objects. It is important that students have multiple opportunities to work with actual objects in the process of measuring.

Domain: **Measurement and Data (MD)**

Cluster: Relate addition and subtraction to length.

Standard: **2.MD.5.** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

Standards for Mathematical Practices (MP) to be emphasized:
MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.8. Look for and express regularity in repeated reasoning.

Connections: **(2.MD.5-6)**
This cluster is connected to the Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction**.

This cluster connects *to Use place value understanding and properties of operations to add and subtract* in Grade 1, to *Represent and solve problems involving addition and subtraction* in Grade 2, and to *Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures* in Grade 3.

Explanations and Examples:
2.MD.5 apply the concept of length to solve addition and subtraction word problems with numbers within 100. Students must use the same unit in these problems.
Example:
In P.E. class Kate jumped 14 inches. Mary jumped 23 inches. How much farther did Mary jump than Kate? Write an equation and then solve the problem.

Student 1
My equation is $14 + \_\_ = 23$ since I am trying to find out the difference between Kate and Mary's jump. I used **place value blocks** and counted out 14. I then added blocks until I got to 23. I needed to add 9 blocks. Mary jumped 9 more inches than Kate.

Student 2
My equation is $23 - 14 = \_\_$. I drew a **number line**. I started at 23. I moved back to 14 and counted how far I moved (the units). I moved back 9 spots. Mary jumped 9 more inches than Kate.

Students need experience working with addition and subtraction to solve word problems which include measures of length. It is important that word problems stay within the same unit of measure. Counting on and/or counting back on a number line will help tie this concept to previous knowledge. Some representations students can use include drawings, number lines, rulers, pictures, and/or physical objects.

Continued next page

An interactive whiteboard or document camera may be used to help students develop and demonstrate their thinking.

Equations include:
- $20 + 35 = c$
- $c - 20 = 35$
- $c - 35 = 20$
- $20 + b = 55$
- $35 + a = 55$
- $55 = a + 35$
- $55 = 20 + b$

Example:
- A word problem for $5 - n = 2$ could be: Mary is making a dress. She has 5 yards of fabric. She uses some of the fabric and has 2 yards left. How many yards did Mary use?

There is a strong connection between this standard and demonstrating fluency of addition and subtraction facts. Addition facts through $10 + 10$ and the related subtraction facts should be included.

Instructional Strategies (2.MD.5-6)

Connect the whole-number units on rulers, yardsticks, meter sticks and measuring tapes to number lines showing whole-number units starting at 0. Use these measuring tools to model different representations for whole-number sums and differences less than or equal to 100 using the numbers 0 to 100.

Use the meter stick to view units of ten (10 cm) and hundred (100 cm), and to skip count by 5s and 10s.

Provide one- and two-step word problems that include different lengths measurement made with the same unit (inches, feet, centimeters, and meters). Students add and subtract within 100 to solve problems for these situations: adding to, taking from, putting together, taking apart, and comparing, and with unknowns in all positions. Students use drawings and write equations with a symbol for the unknown to solve the problems.

Have students represent their addition and subtraction within 100 on a number line. They can use notebook or grid paper to make their own number lines. First have them mark and label a line on paper with whole-number units that are equally spaced and relevant to the addition or subtraction problem. Then have them show the addition or subtraction using curved lines segments above the number line and between the numbers marked on the number line. For $49 + 5$, start at 49 on the line and draw a curve to 50, then continue drawing curves to 54. Drawing the curves or making the "hops" between the numbers will help students focus on a space as the length of a unit and the sum or difference as a length.

49 + 1 + 1 + 1 + 1 + 1 = 54
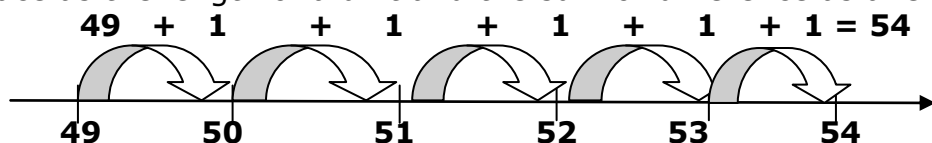
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Instructional Resources/Tools
Rulers
Yardsticks
Meter sticks
Measuring tapes
Cash register tapes or paper strips

Nctm.org Illuminations: Hopping Backward to Solve Problems
In this lesson, students determine differences using the number line to compare lengths.
Nctm.org Illuminations: Where Will I Land?
In this lesson, the students find differences using the number line, a continuous model for subtraction.

2.MD.5

Domain: **Measurement and Data (MD)**

Cluster: Relate addition and subtraction to length.

Standard: **2.MD.6**. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.

Standards for Mathematical Practices (MP) to be emphasized:
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.

Connections:
See 2.MD.5)

Explanations and Examples:
2.MD.6 calls for students to create number lines using numbers within 100 to solve addition and subtraction problems. Students should create the number line with evenly spaced points corresponding to the numbers.

Students represent their thinking when adding and subtracting within 100 by using a number line. An interactive whiteboard or document camera can be used to help students demonstrate their thinking.

Example: 10 – 6 = 4
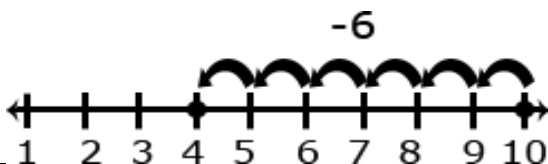
Domain: **Measurement and Data (MD)**
Cluster: Work with time and money**.**
Standard: **2.MD.7.** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
Standards for Mathematical Practices (MP) to be emphasized: MP.5. Use appropriate tools strategically. MP.6. Attend to precision.
Connections: **(2.MD.7-8)** This cluster is connected to the Second Grade Critical Area of Focus #2, **Building fluency with addition and subtraction, and beyond the critical area of focus in addressing, telling time and writing time**. This cluster connects to *Tell and write time in* Grade 1, to *Represent and solve problems involving addition and subtraction* in Grade 2, and to *Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses* of objects in Grade 3.
Explanations and Examples: **2.MD.7** calls for students to tell (orally and in writing) and write time after reading analog and digital clocks. Time should be to 5 minute intervals, and students should also use the terms a.m. and p.m. Teachers should help students make the connection between skip counting by 5s (2.NBT.2) and telling time on an analog clock.   In first grade, students learned to tell time to the nearest hour and half-hour. Students build on this understanding in second grade by skip-counting by 5 to recognize 5-minute intervals on the clock. They need exposure to both digital and analog clocks. It is important that they can recognize time in both formats and communicate their understanding of time using both numbers and language. Common time phrases include the following: quarter till ____, quarter after ____, ten till ____, ten after ____, and half past ____. Students should understand that there are 2 cycles of 12 hours in a day - a.m. and p.m. Recording their daily actions in a journal would be helpful for making real-world connections and understanding the difference between these two cycles. An interactive whiteboard or document camera may be used to help students demonstrate their thinking.   Continued next page

Instructional Strategies (2.MD.7-8)

Second graders expand their work with telling time from analog and digital clocks to the nearest hour or half-hour in Grade 1 to telling time to the nearest five minutes using a.m. and p.m.

The topic of money begins at Grade 2 and builds on the work in other clusters in this and previous grades. Help students learn money concepts and solidify their understanding of other topics by providing activities where students make connections between them. For instance, link the value of a dollar bill as 100 cents to the concept of 100 and counting within 1000. Use play money - nickels, dimes, and dollar bills to skip count by 5s, 10s, and 100s. Reinforce place value concepts with the values of dollar bills, dimes, and pennies.

Students use the context of money to find sums and differences less than or equal to 100 using the numbers 0 to 100. They add and subtract to solve one- and two-step word problems involving money situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. Students use drawings and equations with a symbol for the unknown number to represent the problem. The dollar sign, $, is used for labeling whole-dollar amounts without decimals, such as $29. Students need to learn the relationships between the values of a penny, nickel, dime, quarter and dollar bill.

Common Misconceptions: (2.MD.7-8)

Some students might confuse the hour and minutes hands. For the time of 3:45, they say the time is 9:15. Also, some students name the numeral closest to the hands, regardless of whether this is appropriate. For instance, for the time of 3:45 they say the time is 3:09 or 9:03. Assess students' understanding of the roles of the minute and hour hands and the relationship between them. Provide opportunities for students to experience and measure times to the nearest five minutes and the nearest hour. Have them focus on the movement and features of the hands on real or geared manipulative clocks.

Students might overgeneralize the value of coins when they count them. They might count them as individual objects. Also some students think that the value of a coin is directly related to its size, so the bigger the coin, the more it is worth. Place pictures of a nickel on the top of five-frames that are filled with pictures of pennies. In like manner, attach pictures of dimes and pennies to ten-frames and pictures of quarters to 5 x 5 grids filled with pennies. Have students use these materials to determine the value of a set of coins in cents.

Domain: **Measurement and Data (MD)**
Cluster: Work with time and money**.**
Standard: **2.MD.8.** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*
Standards for Mathematical Practices (MP) to be emphasized: MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision MP.8. Look for and express regularity in repeated reasoning.
Connections: See 2.MD.7)
Explanations and Examples: **2.MD.8** calls for students to solve word problems involving either dollars or cents. **Since students have not been introduced to decimals, problems should either have only dollars or only cents**. **Example:** What are some possible combinations of coins (pennies, nickels, dimes, and quarters) that equal 37 cents? **Example**: What are some possible combinations of dollar bills ($1, $5 and $10) that equal 12 dollars? Since money is not specifically addressed in kindergarten, first grade, or third grade, students should have multiple opportunities to identify, count, recognize, and use coins and bills in and out of context. They should also experience making equivalent amounts using both coins and bills. "Dollar bills" should include denominations up to one hundred ($1.00, $5.00, $10.00, $20.00, $100.00). Students should solve story problems connecting the different representations. These representations may include objects, pictures, charts, tables, words, and/or numbers. Students should communicate their mathematical thinking and justify their answers. An interactive whiteboard or document camera may be used to help students demonstrate and justify their thinking. **Example**: Sandra went to the store and received 76 ¢ in change. What are three different sets of coins she could have received?
Common Misconceptions: Sometimes students will record twenty-nine dollars as 29$. Remind them that the dollar sign goes in front. The cent sign goes after the number and there is no decimal point used with the cent sign.

2.MD.8

Domain: **Measurement and Data (MD)**
Cluster: Represent and interpret data.
Standard: **2.MD.9.** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
Standards for Mathematical Practices (MP) to be emphasized: MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.8. Look for and express regularity in repeated reasoning.
Connections: **(2.MD.9-10)** This cluster goes beyond the Second Grade Critical Area of Focus to address, Using data representations. This cluster connects to *Measure lengths indirectly and by iterating length units* and *Represent and interpret data* in Grade 1, and to *Represent and interpret data* in Grade 3.
Explanations and Examples: **2.MD.9** calls for students to represent the length of several objects by making a line plot. Students should round their lengths to the nearest whole unit. **Example**: Measure objects in your desk to the nearest inch, display data collected on a line plot. How many objects measured 2 inches? 3 inches? Which length had the most number of objects? How do you know?   This standard emphasizes representing data using a line plot. Students will use the measurement skills learned in earlier standards to measure objects. Line plots are first introduced in this grade level. A line plot can be thought of as plotting data on a number line. An interactive whiteboard may be used to create and/or model line plots as well as "Class" line-plots on chart paper.   Continued next page
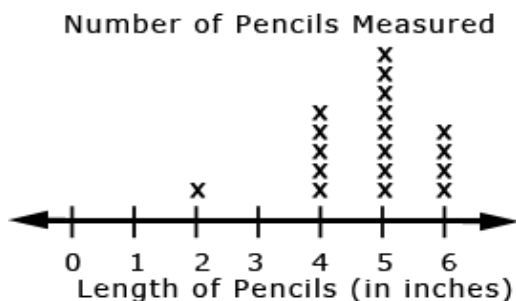
Instructional Strategies: **(2.MD.9-10)**

Line plots are useful tools for collecting data because they show the number of things along a numeric scale. The line plot is made by simply drawing a number line then placing an X above the corresponding value on the line that represents each piece of data. Line plots are essentially bar graphs with a potential bar for each value on the number line.

Pose a question related to the lengths of several objects. Measure the objects to the nearest whole inch, foot, centimeter or meter. Create a line plot with whole-number units (0, 1, 2, ...) on the number line to represent the measurements.

At first students should create real object or picture graphs (where the object is drawn rather than a number). On picture graphs record the number of countable parts. These graphs show items in a category and do not have a numerical scale. For example, a real object graph could show the students' shoes (one shoe per student) lined end to end in horizontal or vertical rows by their color. Students would simply count to find how many shoes are in each row or bar. The graphs should be limited to 2 to 4 rows or bars.

Students would then move to making horizontal or vertical bar graphs with two to four categories and a single-unit scale. Use the information in the graphs to pose and solve simple put together, take-apart, and compare problems illustrated in Table 1 (p.49) of this document.

Common Misconceptions: **(2.MD.9-10)**

The attributes for the same kind of object can vary. This will cause equal values in an object graph to appear unequal. For example, when making an object graph using shoes for boys and girls, five adjacent boy shoes would likely appear longer than five adjacent girl shoes. To standardize the objects, place the objects on the same-sized construction paper or sticky-note, then make the object graph.

2.MD.9

Domain: **Measurement and Data (MD)**

Cluster: Represent and interpret data.

Standard: **2.MD.10.** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (See Table 1.)

Standards for Mathematical Practices (MP) to be emphasized:
MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.8. Look for and express regularity in repeated reasoning.

Connections:
See 2.MD.9

Explanations and Examples:
2.MD.10 calls for students to work with categorical data by organizing, representing and interpreting data.

Students should have experiences posing a question with 4 possible responses and then work with the data that they collect.

Example:
Students pose a question and the 4 possible responses. Which is your favorite flavor of ice cream? Chocolate, vanilla, strawberry, or cherry?
Students collect their data by using tallies or another way of keeping track.
Students organize their data by totaling each category in a chart or table. Picture and bar graphs are introduced in
Second Grade.

Flavor	Number of People
Chocolate	12
Vanilla	5
Strawberry	6
Cherry	9

Students display their data using a picture graph or bar graph using a single unit scale. Students answer simple problems related to addition and subtraction that ask them to put together, take apart, and compare numbers. See Table 1 (p.49) for examples of these.

Students should draw both picture and bar graphs representing data that can be sorted up to four categories using single unit scales (e.g., scales should count by ones). The data should be used to solve put together, take-apart, and compare problems as listed in Table 1, page 49.

In second grade, picture graphs (pictographs) include symbols that represent single units. Pictographs should include a title, categories, category label, key, and data.

Number of Books Read	
Nancy	✧ ✧ ✧ ✧
Juan	✧ ✧ ✧ ✧ ✧ ✧ ✧
✧ = 1 Book	
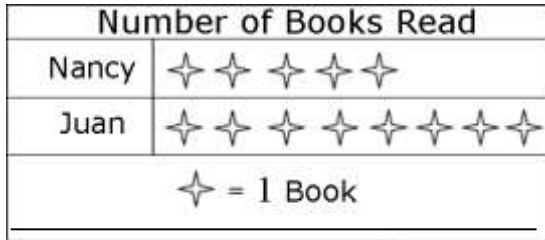
Second graders should draw both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.
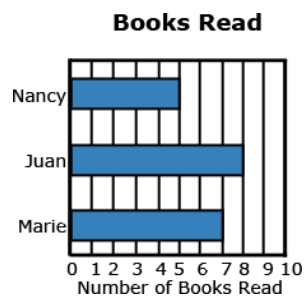
Second Grade Mathematics Extended Standards
Measurement and Data (MD)
(North Carolina DOE)

Common Core State Standards	Essence	Extended Common Core
Measure and estimate lengths in standard units.	**Measure using nonstandard units**	**Measure lengths in non–standard units**
Cluster 1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. 2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. 3. Estimate lengths using units of inches, feet, centimeters, and meters. 4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.		**Cluster** 1. Use nonstandard units to compare length of objects.
Relate addition and subtraction to length	**Relate addition to length**	**Relate addition to length**
Cluster 5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. 6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences on a number line diagram.		**Cluster** 2. Add the number of same units to determine the length of a given object.

Second Grade Mathematics Extended Standards
Measurement and Data (MD)

Common Core State Standards	Essence	Extended Common Core
Work with time and money.	**Use of time and money concepts**	**Work with time and money.**
Cluster 7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. 8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*		**Cluster** 3. Use the names of the days of the week to describe when personal activities will occur. 4. Use a calendar to mark differences between a day and a week. 5. Use a half day schedule to keep track of events with modeling. 6. Solve word problems using one dollar bills or pennies.
Represent and interpret data	**Represent and interpret data**	**Represent and interpret data**
Cluster 9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole number units. 10. Draw a picture graph and a bar graph (with single unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems10 using information presented in a bar graph.		Cluster 7. Organize and represent data using concrete objects to create picture graphs. 8. Interpret collected data to determine the answer to the question posed.

Domain: **Geometry (G)**

Cluster: Reason with shapes and their attributes.

Standard: **2.G.1.** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. *Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

*Sizes are compared directly or visually, not by measuring.

Standards for Mathematical Practices (MP) to be emphasized:
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.

Connections: **(2.G.1-3)**
This cluster is connected to the Second Grade Critical Area of Focus #4**, Describing and analyzing shapes**.

This cluster connects to *Reason with shapes and their attributes* in Grade 1, and to *Develop understanding of fractions as numbers* and *Reason with shapes and their attributes* in Grade 3.

Explanations and Examples:
2.G.1 calls for students to identify (recognize) and draw shapes based on a given set of attributes. These include triangles, quadrilaterals (squares, rectangles, and trapezoids), pentagons, hexagons and cubes.

Example:
Draw a closed shape that has five sides. What is the name of the shape?

Student 1: I drew a shape with 5 sides. It is called a pentagon.

Students identify, describe, and draw triangles, quadrilaterals, pentagons, and hexagons. Pentagons, triangles, and hexagons should appear as both regular (equal sides and equal angles) and irregular. Students recognize all four sided shapes as quadrilaterals. Students use the vocabulary word "angle" in place of "corner" but they do not need to name angle types. Geosticks, geoboards, interactive whiteboards and document cameras may be used to help identify shapes and their attributes. Shapes should be presented in a variety of orientations and configurations.
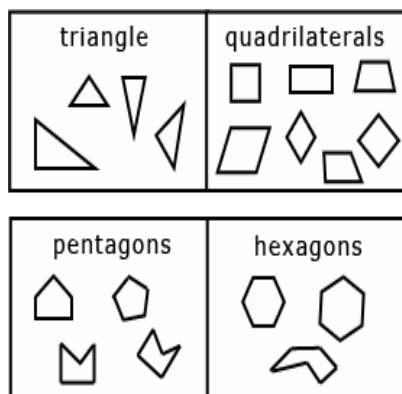

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Instructional Strategies: (2.G.1-3)

Modeling multiplication with partitioned rectangles promotes students' understanding of multiplication. Tell students that they will be drawing a square on grid paper. The length of each side is equal to 2 units. Ask them to guess how many 1 unit by 1 unit squares will be inside this 2 unit by 2 unit square. Students now draw this square and count the 1 by 1 unit squares inside it. They compare this number to their guess. Next, students draw a 2 unit by 3 unit rectangle and count how many 1 unit by 1 unit squares are inside. Now they choose the two dimensions for a rectangle, predict the number of 1 unit by 1 unit squares inside, draw the rectangle, count the number of 1 unit by 1 unit squares inside and compare this number to their guess. Students repeat this process for different-size rectangles. Finally, ask them to what they observed as they worked on the task.

It is vital that students understand different representations of fair shares. Provide a collection of different-size circles and rectangles cut from paper. Ask students to fold some shapes into halves, some into thirds, and some into fourths. They compare the locations of the folds in their shapes as a class and discuss the different representations for the fractional parts. To fold rectangles into thirds, ask students if they have ever seen how letters are folded to be placed in envelopes. Have them fold the paper very carefully to make sure the three parts are the same size. Ask them to discuss why the same process does not work to fold a circle into thirds.

Common Misconceptions: (2.G.1-3)

Some students may think that a shape is changed by its orientation. They may see a rectangle with the longer side as the base, but claim that the same rectangle with the shorter side as the base is a different shape. This is why is it so important to have young students handle shapes and physically feel that the shape does not change regardless of the orientation, as illustrated below.
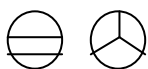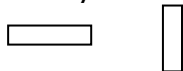
Students also may believe that a region model represents one out of two, three or four fractional parts without regard to the fact that the parts have to be equal shares, e.g., a circle divided by two equally spaced horizontal lines represents three thirds.

Domain: **Geometry (G)**

Cluster: Reason with shapes and their attributes.

Standard: **2.G.2.** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Standards for Mathematical Practices (MP) to be emphasized:
MP.2. Reason abstractly and quantitatively.
MP.6. Attend to precision.
MP.8. Look for and express regularity in repeated reasoning.

Connections:
See 2.G.1

Explanations and Examples:
2.G.2 calls for students to partition a rectangle into squares (or square-like regions) and then determine the total number of squares. This relates to the standard 2.OA.4 where students are arranging objects in an array of rows and columns.

Example:
Split the rectangle into 2 rows and 4 columns. How many small squares did you make?
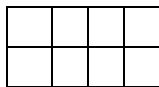
This standard is a precursor to learning about the area of a rectangle and using arrays for multiplication. An interactive whiteboard or manipulatives such as square tiles or other square shaped objects can be used to help students partition rectangles.

Rows are horizontal and columns are vertical.
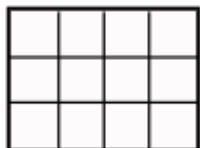
| |
|---|
| Domain: **Geometry (G)** |
| Cluster: Reason with shapes and their attributes. |
| Standard: **2.G.3.** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. |
| **Standards for Mathematical Practices (MP) to be emphasized:**
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.6. Attend to precision.
MP.8. Look for and express regularity in repeated reasoning. |
| **Connections**:
See 2.G.1 |
| **Explanations and Examples:**
2.G.3 calls for students to partition (divide) circles and rectangles into 2, 3 or 4 equal shares (regions). Students should be given ample experiences to explore this concept with paper strips and pictorial representations. Students should also work with the vocabulary terms halves, thirds, half of, third of, and fourth (or quarter) of. While students are working on this standard, teachers should help them to make the connection that a " whole" is composed of two halves, three thirds, or four fourths.

This standard also addresses the idea that equal shares of identical wholes may not have the same shape.

Example:
Divide each rectangle into fourths a different way.
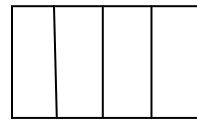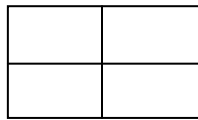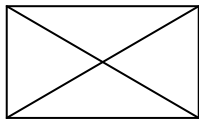
This standard introduces fractions in an **area model**. Students need experiences with different sizes, circles, and rectangles. For example, students should recognize that when they cut a circle into three equal pieces, each piece will equal one third of its original whole. In this case, students should describe the whole as three thirds. If a circle is cut into four equal pieces, each piece will equal one fourth of its original whole and the whole is described as four fourths.
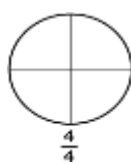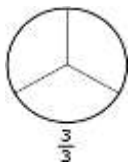
It is important for students to see circles and rectangles partitioned in multiple ways so they learn to recognize that equal shares can be different shapes within the same whole. An interactive whiteboard may be used to show partitions of shapes.

 |

2.G.3

Second Grade Extended Standards
Geometry (2.G)
(NC DOE)

Second Grade Mathematics Geometry (G)			
Common Core State Standards		**Essence**	**Extended Common Core**
Reason with shapes and their attributes		Measure using nonstandard units	**Reason with shapes and their attributes (circles, rectangles, squares and triangles).**
Cluster	1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. 2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. 3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. .		Cluster	1. Use shape names to describe shapes. 2. Match same shapes with different orientation. 3. Identify shapes larger and smaller than model as same shape. 4. Use shapes separately, to make a picture. 5. Match 2 halves of a shape to create whole shape.

EXTENDED 2.G

TABLE 1. Common addition and subtraction situations.[34]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown[35]
Put Together/ Take Apart[36]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, \ 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[37]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, \ 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, \ 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, \ ? + 3 = 5$

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[34] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[35] These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[36] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[37] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.